

# Galois Smartnetwork Field Theory

## Higher-structure symmetry-driven framework for unifying human-AI mathematical discovery across the sciences

Melanie Swan, PhD, MBA, DIYgenomics, University College London (Research Associate CBT); Takashi Kido, PhD, AI and Humanity Program Lead, Common Education Center, Teikyo University, Japan; and Renato P. dos Santos, PhD, academicum.ai, Brazil

### Abstract

Higher-structure mathematics is emerging as a shared conceptual frontier in which biological and artificial systems converge. Mathematical creativity is shifting from incremental reasoning toward a broader form of human-AI inquiry in mathematics and the sciences, with research-level mathematics becoming the new benchmark for AI capability. Entering an era of AI-accelerated digital mathematics demands new theory-level resources to guide progress. Galois Smartnetwork Field Theory (Galois SNFT) is elaborated as a unifying framework which integrates mathematics, computation, and physics through higher structure principles, especially symmetry. The approach offers a principled blueprint for human-AI collaboration in novel discovery across the sciences.

### Galois Smartnetwork Field Theory

**Galois (math) Smartnetwork (computation) Field Theory (physics)**  
**Galois:** (1) Galois formalisms, including contemporary extensions (higher-form symmetries, Galois categories, étale topology) (2) shorthand for symmetry or foundational physics principles (3) method to detect hidden information in higher dimensions  
**Galois's idea:** identify solvable equations (with  $x^2$ ) by looking at the permutations of the symmetry properties of roots (invented **group**)  
**Smartnetwork:** modern computational infrastructure: AI, quantum, blockchains, robotics, neurotech, cloud, AI datacenters, digital twins  
**Field theory:** core formalization in both mathematics and physics

**Background:** Galois SNFT extends NNFT (Neural Network Field Theory), field-theoretic formulations of NN with QFT since NNs and QFT both describe probability distributions over functions (fields), by joining statistics and dynamics (Figure 1).

Figure 1. Background of Galois Smartnetwork Field Theory in Neural and Statistical Field Theories

Topic	Description	Reference	Mathematics	Computation	Physics
<b>Galois SNFT Formulations (Research-math NNFTs)</b>					
1	NNQFT	Transformer attention heads as symmetry-breaking	2026, Swanson [1]; 2026, Ageev [2]	2024, Hattar et al. [3]	
2	INFT	Fisher information-theoretic line-tuning	2026, Halverson et al. [4]		
3	KTNN	Categorical info-flow-matching (Fisher, Ricci, Riemann)	2026, 2025, Bronstein et al. [5,6]		
4	KTHN	Knot-theoretic NN-ribbon disk bounding & Ricci flow	2025, 2024, Gukov et al. [7,8]		
5	STNN	Sheaf-theoretic NN: poset sheaf cohomology	2025, Gebhart et al. [9]		
<b>Foundational Formulations (NNFTs)</b>					
1	NNFT	Proof: any QFT has a NN description	2026, Swanson et al. [10]		
2	NNQFT	Gaussian processes and tangent kernels	2021, Halverson et al. [11]; 2021, NEY REF		
3	SNFT	Statistical field theory for NN	2019, Helke & Dahmen [12]		
4	SNFT	Smartnetwork field theory	2018, 2020, Swan et al. [13,14]		
5	NSFT	Neural statistical field theory	2007, Bucci & Cowan [15]		

**Implementation:** Galois-type quantum circuits realize **Quantum Mathematics** in AI-Quantum Networks (the learning capacity of AI coupled with the compute muscle of quantum) (Figure 2).

Figure 2. Quantum Mathematics instantiated in Galois-type Quantum Circuits

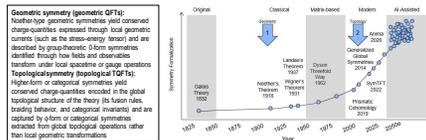
Description	Quantum Math	Quantum App	Reference
1	Symmetry discretizations	Q-CIRCUITS	2026, Preskill et al. [1]
2	Hybrid lattice surgery	QEC	2026, Schafner-Nemethy [2]
3	Transversal gates	DECODING hypergraphs	2025, Lakin et al. [3]
4	Quantum state tomography	ADAPTIVE algorithms	2025, Eisen et al. [4]
5	Quantum entangled spaces	Cohomology obstruction	2026, Heide & Rayan [5]
6	Energy teleportation	Entanglement geometry	Q TELEPORT 2026, 2025, Seca [6,7]
7	Time entanglement	Correlation functions	Entanglement 2025, Preskill et al. [8]
8	Time entanglement	Knot invariants	Entanglement 2025, Brubaker et al. [9]
9	Time series analysis	Ribonovan homology	QFN 2025, Lloyd et al. [10]
10	Quantum tensor	Hoop algebras	QSM 2026, Lloyd et al. [11]
11	Knot polynomial	Knot invariants	QALG/QSM 2024, Gao et al. [12]
12	Primot gas	Automorphic L-functions	Riemann zeros 2025, Hartnell et al. [13]
13	Non-hermitian crystals	Hyperuniformity	Riemann zeros 2026, 2024, Park [14,15]

QEC: Quantum Error Correction, QMCM: Quantum Memory, QFN: Quantum Finance, QSM: Quantum Simulation, QALG: Quantum Algebra

**Mathematics:** Geometry and Topology both investigate shapes and spaces but in different ways  
• **Geometry (mathematics):** measurement-based, concrete total metric of least angles, distance, length, volume, curvatures  
• **Topology (physics):** abstract, structure-based, global invariance with space-embedding, closeness, continuity, connectivity, holes  
• **Homology (mathematics):** the flexible framework of mapping low shape without adding or gluing  
• **Homology (physics):** the structural differences of holes, voids, holes, and gaps that survive deformation  
Use Case: Identify fusion invariants and whether high-dimensional spaces are the same (robotic world-model building, medical imaging, QFT)

### Modern Symmetry Theory

Figure 3. 200 Years of 'Moore's Law of Symmetry'



Formalism	Field	Math	Formalism	Math	Formalism
1. QFT	Geometry	Group Theory	0-form symmetry	Local	Physics (rotation, reflection, displacement, conformal, gauge)
2. TQFT	Topology	Category Theory	0-form symmetries	Global	Higher-structure: robust noise, leading behavior, categorical invariants

**Moore's Law of Symmetry:** Advances in symmetry began with Galois and ensue today with the two major findings of geometric symmetry (QFTs) and topological symmetry (TQFTs) (Figure 3). Three contemporary theories constitute Galois SNFT (Figure 4).  
**1. Anima:** (Greek: *soul; animating principle*): core shape (homotopy type) of a condensed or highly structured object, extracted by classifying symmetry and descent data into an infinity-groupoid (a more flexible group including its elements and all their connections)  
**2. Prismatic cohomology:** invariants assigned to a topological space to study symmetry properties; flavors of cohomology (e.g.: Galois representations) are unified as different views of a prism  
**3. SymTFT:** (Symmetry Topological Field Theory): generalized global symmetries encoded as a TFT in one higher dimension to treat system-level of topological defects, anomaly, non-invertibility. These theories reflect the trend of lifting **groups** to **categories**, foregrounding **relational** structure over object content.

Figure 4. Modern Symmetry Theory developed into Galois SNFT Symmetry Principles

Layer	Function	Modern symmetry theory tools	Mathematical analogies	Galois SNFT building blocks	
1	Ontology (space)	The definition of the symmetry-world	Anima	Homotopy (deformation)	Global propagation
2	Geometry (coordinates)	The data that lives in this world	Prismatic cohomology	Cohomology (invariant)	Symmetry constraint
3	Dynamics (evolution)	How symmetries act in this world	SymTFT	Field Theory (anomalies)	Phase stability

**Galois SNFT:** In this structural backbone, a **Galois anima** provide the fundamental symmetry shape, **prismatic cohomology** supplies the unified arithmetic data that the symmetry acts on, and **SymTFT** encodes the higher categorical rules governing how these symmetries behave, interact, and classify phase transitions.

**1. Global propagation → Anima**  
Local symmetry data must glue into one global symmetry-shape. The Galois anima captures this by encoding the resulting homotopy-type (shape) that emerges when all local Galois relations cohere into a single global object. Galois SNFT's *ontological law*.

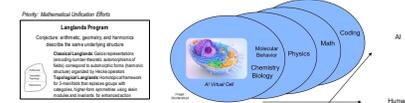
**2. Symmetry constraint → Prismatic cohomology**  
Only structures compatible with the governing symmetry patterns are allowed. Prismatic cohomology enforces this by packaging étale (Galois), de Rham, and crystalline data into one system of symmetry-consistent invariants. Galois SNFT's *geometric law*.

**3. Phase stability → SymTFT**  
Key invariants stay fixed under continuous deformation until a phase boundary is crossed. SymTFT models this by treating the anima and its prismatic invariants as symmetry-protected phases whose behavior is governed by topological stability rules. Galois SNFT's *dynamical law*.

### Application: Millennium Prize Problems

**Era of Human-AI Math Collaboration.** Math Agents are shifting from contest-math (consistent gold-medal performance) to research-math. Verification of results is the bottleneck. Mathematicians publish open problems to internet-based collaborations and speak natural language (vibe-math) with LLMs to formalize Lean proofs (formal language). Articulating biological breakthroughs is difficult.

Figure 5. Human-AI Collaboration Progress in Software, Mathematics, and the Sciences



**Millennium Prize Problems (MPP):** MPP are seven mathematics problems whose resolution is thought to require the discovery of new kinds of mathematics. The Poincaré Conjecture was solved in 2003 with a novel Ricci flow lattice surgery method. Here the MPP are organized in the schema of three Langlands problems (pure mathematics problems based in algebra and harmonics), three geometric flows problems (physics problems formulated with algebra), and one orthogonal problem in computation theory (Figure 5). A 2023 prediction-market estimate of MPP resolution may be altered by new techniques of AI solvability (Figure 6). Navier-Stokes is an announced target of top AI research labs.

Figure 6. MPP Arms: Langlands, Physics, and Orthogonal, and Prediction-Market Estimates (Metaculus)

Millennium Prize Problem	Description	AI Impact
<b>Langlands Arm (algebra-spectrum relation)</b>		
1. Riemann Hypothesis (RH)	Distribution of prime numbers follows a spectral line?	HIGH
2. Birch-Swinnerton-Dyer (BSD)	Algebraic signal strength predicts curve complexity?	HIGH
3. Hodge Conjecture (HC)	Algebraic structure produces geometric shapes?	HIGH
<b>Geometric Flows Arm (algebra-physics relation)</b>		
4. Poincaré Conjecture (PC)	3D shapes without holes same as stretched 3-sphere?	SOLVED
5. Yang-Mills and Mass Gap (YM)	Why do quantum fields have a minimal energy?	LOW
6. Navier-Stokes Existence and Smoothness (NS)	Is truly smooth fluid motion always singularities?	HIGH
<b>Orthogonal Arm (computation theory)</b>		
7. P vs NP (P/NP)	Problems with quick verification also quickly solvable?	LOW

**METHOD:** Apply Galois-SNFT to MPP. Galois SNFT is most-suited to the Langlands arm of the MPP as the problems hinge on symmetry-driven correspondence, for example RH. RH says that the hidden vibration pattern underlying the distribution of prime numbers follows a perfectly balanced symmetry, with all its nontrivial frequencies lining up on a single critical line. In Galois SNFT, RH becomes the single symmetry-stability claim that the global Galois anima of the spectrum of integers admits only one symmetry-consistent phase, and in that phase the zeta spectrum is forced to lie on the critical line.

**RESULT:** Next steps toward a demonstration plan are identified:  
**1. Anima layer:** Build a single symmetry-shape for primes by gluing all local zeta and Galois data into one global anima, then interpret "zeros on the critical line" as a constraint on anima symmetry-type  
**2. Prismatic layer:** Encode zeta's spectral behavior in one unified prismatic invariant, turning RH into symmetry-balanced eigenvalue condition inside the prismatic cohomology of the zeta anima  
**3. SymTFT layer:** Treat possible zero distributions as phases of a symmetry field theory and use anomaly constraints to show that only the RH phase is dynamically stable

### Application: Longevity Science

A crucial target for AI collaboration is humanity-facing problems such as Longevity Science. Higher-level mathematical modeling is needed for an improved understanding of the molecular factors underpinning longevity, healthy aging, and age-related disease. Three core topics in Longevity Science are gene regulatory network (GRN) changes, epigenetic drift, and cellular repair.  
**Gene regulatory networks:** interacting control circuits that turn genes on and off; typically modeled with ODEs, Boolean dynamics  
**Epigenetic drift:** gradual random alteration of chromatin marks that destabilize cell identity; modeled with SDEs, random walks, entropy  
**Cellular repair:** processes cells use to detect and fix molecular damage; modeled with Markov chains, queueing, enzyme kinetics

**METHOD:** A Galois-SNFT approach is applied to GRNs, epigenetic drift, and cellular repair, first identifying these higher-math themes:  
• **GRNs** show network patterns of regulation and feedback that break symmetry across a high-dimensional state space  
• **Epigenetic drift** represents a probability landscape in a high-dimensional stochastic-geometry space  
• **Cellular repair** indicates nonlinear, multi-phase transitions as cells move from damage to response to resolution  
The Galois-SNFT framework then interprets the three processes a shared structure in which **anima** capture their underlying symmetry-shapes, **prismatic-style invariants** unify their heterogeneous data into a single constraint-compatible system, and **SymTFT** models their phase-stability and defect-removal dynamics.

Figure 7. Longevity Science Problem amenability to Galois-SNFT Approach (Scoring: 1 = low fit, 5 = high fit)

Longevity Science Topic	Anima (symmetry shape)	Prismatic style invariants	SymTFT (phase/dynamics)
1. Gene regulatory networks	5	4	5
2. Epigenetic landscapes	4	5	4
3. Cellular repair	3	3	5

**RESULT:** Galois SNFT is best-suited to Epigenetics with overall symmetry formulations; cell repair as phase transition (Figure 7). **GRNs:** top score in anima and SymTFT, and strong prismatic fit suggest higher-order symmetry-shapes, admit unified multi-layer invariants, and display clear symmetry-protected dynamical phases  
**Epigenetic drift:** high prismatic and strong anima/SymTFT scores capture epigenetic state spaces with topological structure, unify chromatin marks into shared invariants, and cell identity phases  
**Cellular repair:** moderate anima/prismatic but maximal SymTFT scores show that repair is less symmetry-structured at the shape and invariant level but may be dominated by sharp, stability governed phase transitions from damage to response to resolution

### Conclusion

The development of AI mathematical infrastructure demands new modes of mathematical practice to address complex real-world problems. Galois Smartnetwork Field Theory is discussed as an integrated mathematics, computation, and physics framework for human-AI collaboration, with illustrations in mathematics and longevity science. Research-mathematics has become essential to AI foundation models. Symmetry (invariance) is further underlined as a marquis principle for the transmission of ideal contents.